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Universality

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Abbreviations

ETAS	Epidemic type aftershock (model)
RG	Renormalization group
SOC	Self-organized criticality

Definition

The term universality originates in the theory of phase transitions and self-organized criticality. Phase transitions represent changes between different states of matter which occur when a control parameter (e.g., temperature) attains a critical threshold. The correlations of relevant system properties near a phase transition do not fall off exponentially, and thus they do not exhibit characteristic length scales. In contrast, the decay of the correlations follows power-law functions characterized by respective critical exponents. The theory of self-organized criticality posits that certain systems (e.g., sandpiles, avalanches, and geological faults in seismogenous zones) are close to a critical threshold without tuning a control parameter such as temperature. The term "universality" indicates that phase transitions which occur in different systems are described by the same set of critical exponents, if they share certain geometric and symmetry features; systems with the same values of critical exponents belong in the same universality class (Kadanoff 2000). In a broader sense, universality refers to empirical laws that have global validity, e.g., Gutenberg-Richter's law which relates the frequency and intensity of earthquakes. The exponent of Gutenberg-Richter's law surprisingly takes values close to one regardless of geographical location, geological factors, and seismic history.

Overview

The term universality appears in the theories of phase transitions and self-organized criticality (Bak 2013; Kadanoff 2000). The main feature of systems close to a phase transition is that their response functions lack characteristic length scales. Instead, the correlations in such systems decay as power laws, i.e., scale-free functions determined by respective power-law exponents. The concept of universality posits that systems which are quite different in their microscopic details can exhibit identical, i.e., *universal*, macroscopic behavior. More specifically, universality refers to the fact that the power-law exponents, which describe the macroscopic response of different critical systems belonging in the same universality class, take identical values.

The emergence of universality is often justified in terms of the renormalization group (RG) theory introduced by Wilson (1971). RG provides a systematic mathematical framework for predicting the behavior of physical systems (or idealized mathematical models) under a sequence of coarse-graining transformations. These progressively increase the length scale of observation (coarse-graining), while the short-scale details are effaced; at the same time, the impact of coarse-graining on the system parameters is calculated, and their values are accordingly adjusted. Under repeated RG steps, the system "flows" toward limiting states which are known as fixed points. The defining feature of the latter is that a system remains invariant under further coarse-graining transformations at a fixed point.

Phase transitions often involve a jump between a less ordered and a more ordered state. As the transition threshold is approached from the side of the less ordered state, the system is attracted toward a nontrivial fixed point which determines the properties of the more ordered state and the

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values of its characteristic critical exponents. A universality class encompasses all the models that converge to the same fixed point under application of the RG procedure. Hence, in a broader sense, the term universality refers to the fact that identical macroscopic physical laws apply to systems which may vary in their microscopic details. For example, the laws of thermodynamics do not depend on the details of atomic interactions, the statistical properties of turbulence in the inertial range are independent of the dissipation mechanisms, and the effective permeability of statistically isotropic random media is insensitive to the exact form of the local permeability correlations. In this sense, universality is a main reason for the success of the scientific approach, since it enables a general understanding of natural phenomena based on a few fundamental physical principles.

More specifically, in the context of geosciences, the term universality is used to imply the global validity of an empirical law across different geographical areas and times. For example, the Gutenberg-Richter law of seismicity is considered a universal law since its validity has been confirmed by countless observations and analysis of seismic catalogs worldwide. The exponent of Gutenberg-Richter's law takes a value close to one, although the exact value may not be universal. Note that strictly speaking, the term "universality" implies not only the same behavior (i.e., power-law), but more importantly also common critical (power-law) exponents for different systems that belong in the same universality class.

The concept of universality is also linked with fractal geometry. The fractal approach was developed and popularized by Benoit Mandelbrot (e.g., as described in his book *The Fractal Geometry of Nature*). Fractal patterns consist of *self-similar* structures which lack characteristic length scales. For example, a fractal object of radius r has surface area $S(r) \sim r^{\alpha}$ where $\alpha \neq 2$ is a noninteger (fractal) exponent. In phase transitions, fractal patterns (e.g., clusters of the ordered phase) emerge in systems which are close to the critical state. Fractal patterns in systems that belong in the same universality class share the same fractal exponents.

All notions of universality discussed above have applications in the geosciences. Given the wide range of applications and space limitations, it is not possible to cite important early papers by Leo Kadanoff and Per Bak. The interested readers can find more information about these works in the books by Kadanoff (2000) and Bak (2013).

Universality in Phase Transitions

Phase transitions are mechanisms that enable changes between different states of matter, e.g., water, vapor, and ice. Thus, phase transitions play a prominent role in the water cycle of the Earth. There are many types of phase transitions. Some are controlled by temperature (i.e., thermodynamic transitions), while others by the geometry of the medium in which the process takes place (i.e., geometrical transitions). An example in the first class is the superconducting transition: Metals and ceramic oxides start to conduct electricity without any resistance if the temperature is lowered below a certain critical threshold. An example in the second class is *percolation*: A random porous medium is permeable to fluids if its porosity exceeds a critical threshold. Percolation is key to the production of an aromatic cup of filtered coffee as well as to the infiltration of rainwater into the ground.

Phase transitions are classified as first order and second order. In first-order transitions, the change of state is accompanied by the absorption or release of latent heat, which changes discontinuously the thermodynamic free energy of the system. This leads to a discontinuity in the first derivative of the free energy with respect to a thermodynamic variable (hence the term "first-order"). Typical examples include the condensation of vapor into water droplets (which involves latent heat release) and the melting of ice (mediated by latent heat absorption). In second-order phase transitions, on the other hand, the free energy changes continuously. The discontinuity appears in the susceptibility which is the second-order derivative of the free energy. A typical example is the transition between paramagnetism and ferromagnetism in magnetic materials that takes place at the Curie point. The magnetization of the material acts as the order parameter: It is zero in the paramagnetic phase and takes a finite value in the ferromagnetic state. Similarly, order parameters are defined for other second-order phase transitions (Kadanoff 2000).

Second-order phase transitions are characterized by a set of *critical exponents* that determine how relevant properties scale near the transition point. For example, the exponent ν describes the divergence of the fluctuations' correlation length, i.e., $\xi \sim |T - T_c|^{-\nu}$, where T is the temperature, T_c is the critical temperature, and ξ is the correlation length. The exponent η describes the power-law decay of the correlations with distance at the transition point, i.e., $C(r) \sim r^{-d+2-\eta}$. The exponents α, β, γ, and δ describe the dependence of: (i) the specific heat $c \sim |T - T_c|^{-\alpha}$, (ii) the order parameter $\phi \sim |T - T_c|^{\beta}$, (iii) the susceptibility of the order parameter to an external field h, i.e., $d\phi/dh \sim |T - T_c|^{-\gamma}$, and (iv) the dependence of the external field on the order parameter $h \sim \phi^{\delta}$. The critical exponents depend on the dimensionality of the system and the symmetry of the order parameter, but they may be independent of other details (e.g., lattice structure).

Percolation, Fractals, and Multifractals

Percolation is a geometric phase transition which takes place as a connected (e.g., permeable) structure grows in a random medium until it spans the entire extent of the medium. For example, this happens on a lattice system as the proportion of connected sites (e.g., the porosity) approaches a critical threshold. There exist different percolation models which include site, bond, and continuum percolation (Isichenko 1992). Percolation can be used as a mathematical model for disordered media including heterogeneous geological porous media. The geometric properties of percolation can be applied to analyze transport in such media. Hence, percolation is an important model for the study of subsurface flow and contaminant transport, especially in highly heterogeneous and fractured soils.

Similarly to other phase transitions, percolation involves a critical threshold p_c. This corresponds to the occupation probability above which a fully connected network (medium) becomes possible. The onset of percolation is marked by the emergence of an incipient infinite cluster (essentially a “giant component”) that extends across the medium. The order parameter is given by the probability that a site belongs to this giant component. Other variables describe geometrical patterns of the percolation network (e.g., size of connected clusters, average radius of clusters of given size, probability that two sites at a given distance belong to the same cluster, etc.). Near the percolation threshold, the behavior of such variables is characterized by critical exponents. Universality in the context of percolation means that the critical exponents depend on the dimension of space in which the percolation model is embedded, but they are independent of the lattice structure and the percolation type (site and bond percolation are in the same universality class).

The critical behavior of percolation implies a fractal geometry for the network patterns (e.g., connected clusters) near the percolation threshold. The fractality is bestowed by the critical exponents which in general take noninteger values. The concepts of universality and scaling have found applications in the geometry of naturally occurring fractal patterns such as river networks, fault systems, coastlines, and mountain topography (Isichenko 1992; Dodds and Rothman 2000).

An extended notion of universality is also used in the theory of multifractals. These mathematical models find applications in natural phenomena such as clouds, rain, fluid turbulence, and weather (Lovejoy and Schertzer 2013). Multifractals have more complex scaling relations than fractals. The fractal exponents (dimensions) of multifractals depend on location, and thus a spectrum of exponents is necessary to fully describe multifractal behavior.

Self-Organized Criticality and Earthquakes

A notable application of the concept of universality is the investigation of avalanches in sandpile models by Leo Kadanoff and coworkers in a paper published in 1989 (c.f. Kadanoff 2000). They found different universality classes based on the microscopic rules used for the generation of avalanches. Their work paved the way for the development of the theory of self-organized criticality (SOC), which describes systems that are always close to a critical threshold without tuning of an external parameter (e.g., temperature).

Self-organized criticality has been proposed as a theoretical framework that among other things can explain the laws of seismicity. In particular, the concept of self-organized criticality provides a possible explanation for the Gutenberg-Richter law which connects the frequency and magnitude of earthquake events (Bak 2013). The fact that earthquakes all over the globe follow the Gutenberg-Richter law is seen as a signature of universality. An influential paper by Olami et al. (1992) argues that the Burridge-Knopoff spring-block model used to simulate earthquakes exhibits *self-organized criticality* but lacks *detailed universality*. This means that the exponent of the Gutenberg-Richter law depends on the elastic parameters of the model (Olami et al. 1992). Recent numerical experiments and lab-generated quakes also show a dependence of the Gutenberg-Richter exponent on the material properties of the medium in which the events take place.

According to a different theoretical approach, the probability distribution of *earthquake recurrence times* follows a universal law, provided that the time between earthquakes is scaled by the local rate of seismic occurrence (Corral 2004). However, this viewpoint is controversial; other researchers argue in favor of an almost-universal law based on the epidemic type aftershock (ETAS) model of seismicity (e.g., Saichev and Sornette 2007).

Summary and Conclusions

The term universality appears in different theories of natural phenomena (phase transitions, self-organized criticality, fractals, and multifractals), and its meaning may differ in each case. The general idea of universality is that certain “universal properties” depend on a few “macroscopic” parameters but are independent of the specific details. A common signature of universality is the presence of power-law dependence, often when a critical threshold is approached. The connection between power-law functions and universality is reviewed in Marković and Gros (2014). Overall, universality is a very fruitful concept which allows studying seemingly disparate systems in the same theoretical framework and has several applications in the geosciences.

Cross-References

► Fractals
► Gutenberg-Richter

► Multifractals
► Power Laws
► Scaling

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