A Monte Carlo approach for improved estimation of groundwater level spatial variability in poorly gauged basins

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1. INTRODUCTION

Geostatistical methods in association with auxiliary information in groundwater level applications provide:

- Improved space-time visualization of the aquifer free surface
- Maximization of the information gain for the quantification of groundwater level spatial variability.

Such an application is implemented for a poorly gauged basin on the island of Crete (Greece). The level at 10 wells (green) is monitored between the years 1981 and 2003 bi-annually and in 70 locations (red) for the wet season of 2003.

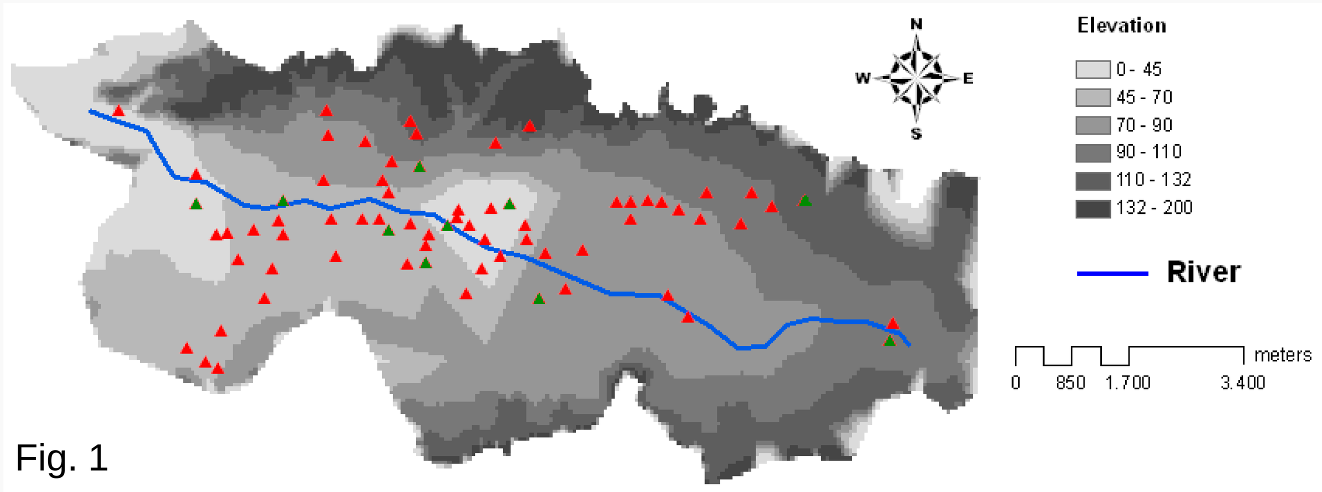


Fig. 1

2. MATERIALS & METHODS

Physical laws incorporation in stochastic hydrology aspects and in classical geostatistics gave the idea to introduce a spatial trend model that could incorporate in the trend a physical law that describes the basin's aquifer behavior with respect to groundwater level and pumping activity.

- **Spatial variation:** We include in the trend modeling the analytical solution for a system of multiple wells in an unconfined aquifer. This component of the trend is based on Thiem's equation for an unconfined aquifer. The equation describes the relationship between the steady-state radial inflow into a pumping well and the drawdown.

$$H^2(\mathbf{s}) = H_0^2(\mathbf{s}) + \frac{1}{\pi K}\sum_{i=1}^{n} Q_i \ln\left(\frac{r_i}{R_i}\right), \quad r_i < R_i, \quad i=1\ldots,n \qquad (1)$$

$H(\mathbf{s})$ is the estimated hydraulic head, $H_0(\mathbf{s})$ is the initial hydraulic head before abstraction, K is the hydraulic conductivity, n is the number of wells (i=1…,n), $r_i = \| \mathbf{s} - \mathbf{s}_i \|$ is the distance of the estimation point from the i-th well, Q is the pumping rate, and R_i is the well's radius of influence. The pumping wells contributing in the equation are those whose distance from the estimation point does not exceed their radius of influence. When pumping tests are not available, R_i is determined using empirical equations, subject to available hydrogeological field data, i.e.,

$$R_i = 575\, s_{w,i} \sqrt{\hat{H}_{0,i}\, K_i}$$

where $s_{w,i}$ is the drawdown at the well face (m), K is the hydraulic conductivity around the pumping well and $\hat{H}_{0,i}$ (m) is the initial saturated thickness.

These variables are not known at every well so uniform values are used. The optimal $s_{w,i}$ is determined from a Linear regression analysis of the mean annual groundwater levels for a 30 year period. The rate of mean annual level decrease is estimated at 1.85m/yr, with the 95% confidence interval at [1.60 - 2.10]. The optimal K is determined by means of a Monte Carlo sensitivity analysis that focuses on the reproduction of the measured head values by means of leave-one-out cross validation and Residual Kriging **(**RK). The mean absolute error (MAE) is used as the criterion of performance. The hydraulic conductivity is sampled from a uniform distribution that extends between the minimum and maximum values, 0.00014 to 0.0014 m/s, for the area. The drawdown values are sampled from the uniform probability distribution over the 95% confidence interval. The hydraulic head trend function is then estimated for each combination of $s_{w,i}$ and K. The average pumping rates (m^3/h) at the 70 wells of the study area are used.

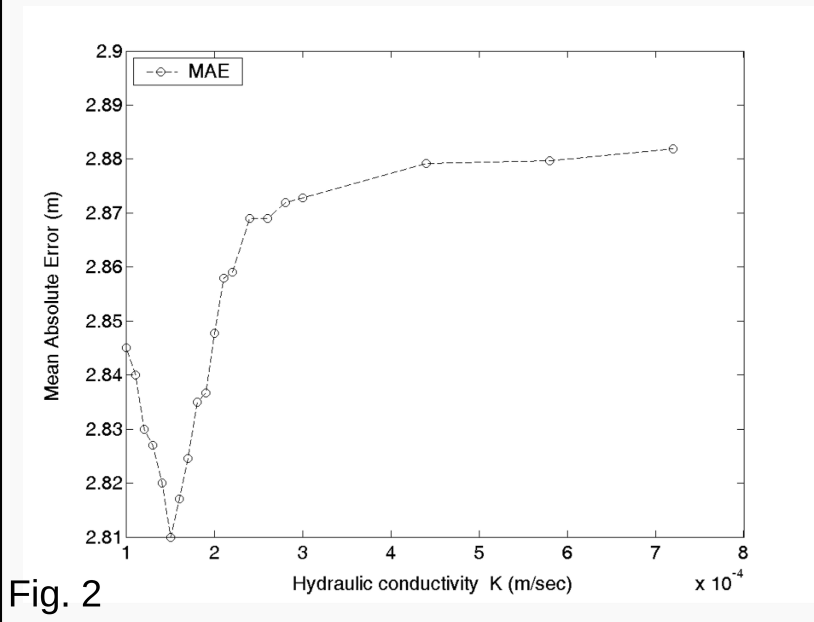


Fig. 2

These Monte Carlo simulations show that the MAE is primarily sensitive to the variation of the hydraulic conductivity. Fig. 2 demonstrates the dependence of the MAE on the hydraulic conductivity: a clear minimum is obtained for K = 0.00015 m/s.

The most accurate results were produced using the flexible Spartan variogram family. Then we can proceed and perform spatial interpolation of the groundwater level in the basin using RK: combining the trend function with interpolation of the residuals .

$$C_z(\mathbf{h}) = \begin{cases} \dfrac{\eta_0\, e^{-h\beta_2}}{2\pi\sqrt{|\eta_1^2-4|}}\left[\dfrac{\sin(h\beta_1)}{h\beta_1}\right], & \text{for } |\eta_1|<2,\ \sigma_z^2 = \dfrac{\eta_0}{2\pi\sqrt{|\eta_1^2-4|}} \\ \dfrac{\eta_0\, e^{-h}}{8\pi}, & \text{for } \eta_1 = 2,\ \sigma_z^2 = \dfrac{\eta_0}{8\pi} \\ \dfrac{\eta_0\,(e^{-h\omega_1}-e^{-h\omega_2})}{4\pi(\omega_2-\omega_1)\,h\sqrt{|\eta_1^2-4|}}, & \text{for } \eta_1 > 2,\ \sigma_z^2 = \dfrac{\eta_0}{4\pi\sqrt{|\eta_1^2-4|}} \end{cases} \qquad (2)$$

where η_0 is the scale factor, η_1 is the rigidity coefficient, β_1 is a dimensionless wavenumber, β_2 and $\omega_{1,2}$, are dimensionless damping coefficients, ξ is a characteristic length, $\mathbf{h}$ = $\mathbf{r}/\xi$ is the normalized lag vector, $h = |\mathbf{h}|$ is its Euclidean norm and σ_z^2 is the variance.

- **Spatiotemporal variation:** We can also proceed to perform spatiotemporal interpolation, approximating the spatiotemporal trend of the field data by multiplying the spatial and the temporal trend (3). The first component is the temporal trend, which is approximated by applying an exponentially-weighted moving average filter in the mean annual groundwater levels, and the second is the spatial trend for the year 2003, which is approximated using Eq. 1.

$$m_z(s,t) = \hat{m}_z(t) \times m_z(s), \quad \hat{m}_z(t_i) = \alpha z(t_i) + (1-\alpha)\hat{m}_z(t_{i-1}) \qquad (3)$$

m is the trend and $0 < \alpha \le 1$ is the weight of the temporal model.

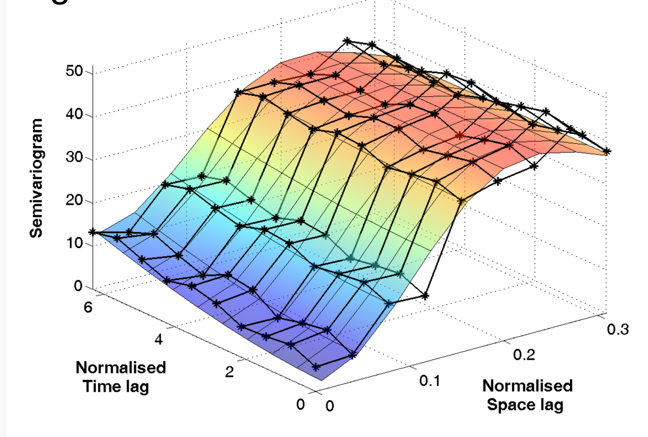
Then, we apply a spatiotemporal geostatistical analysis of the residuals using space-time Residual Kriging (STRK). We model the variogram of the residuals with the new non-separable theoretical spatiotemporal variogram function, which is based on the Spartan variogram family. The function is derived by substituting h with the following equation in its spatial form (2),

$$\mathbf{h} = \sqrt{\mathbf{h}_r^2 + \alpha\,\mathbf{h}_\tau^2}, \quad \mathbf{h}_r = \frac{r}{\xi}, \quad \mathbf{h}_\tau = \frac{\tau}{\xi} \qquad (4)$$

STRK combines the spatiotemporal trend with the interpolation of the residuals in the desired location in order to calculate the groundwater level in 5 measured locations for time periods after 2003. Thus the proposed model is validated.

3. RESULTS

Fig. 3



2004 Wet period highest raise		2006 Dry period highest decrease	
Well No	AE (m)	Well No	AE (m)
G1	2.56	G1	2.49
G2	3.05	G2	3.12
G3	2.05	G3	2.97
G4	2.08	G4	2.18
G5	1.56	G5	0.86

4. CONCLUSIONS

- Monte Carlo approaches can provide valuable information in applications with limited data availability and improve estimations
- The novel Spartan family-based spatiotemporal covariance function modeled the data with higher accuracy than a non-separable function, based on the diffusion equation, overall by 20%
- The incorporation of a physical law in the trend provided an efficient tool for spatial and for spatiotemporal interpolation.

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